Participating insurance contracts and the Rothschild-Stiglitz equilibrium puzzle

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July 2009

Overview

1. Motivation and model
2. Market equilibrium with participating contracts.
3. Deferred premium variations.
4. Concluding comments

Motivation

- The RS (1976) model : one of the most influential contributions in the insurance economics litterature.
- An enigma : no equilibrium exists when second-best efficiency requires cross-subdization between contracts...
- ... which stimulated a lot of research: Wilson (1977), Miyazaki (1977), Spence (1977), Riley (1979), Hellwig (1987), Engers and Fernandez (1987), in a dynamic setting.
- Problem with these models: the timing is very arbitrary... which limits the relevance of the conclusions.

- This paper heads in a different direction: focus on the nature of insurance contracts. RS restrict attention to non-participating insurance contracts. There is no ground (neither theoretical nor empirical) for such a restriction.

- Insurers may also offer participating contracts (i.e. contracts with policy dividends or supplementary call). In the real world mutuals (and sometimes stock insurers) offer participating contracts. The mutual market share at the end of 2006 was 28% for non-life business worldwide (40% in Germany, 40% in France, 36% in Japan, 30% in USA...).

- Transferring underwriting profits to reserves and increasing or decreasing premiums according to the level of accumulated surplus may act as a substitute to policy dividend or supplementary call

- A few papers have adressed the role of mutuals offering participating contracts in the RS environnement: Boyd, Presscott and Smith (1988), Smith and Stutzer (1990), Ligon and Thistle (2005)... but without focusing attention on the equilibrium existence issue and on the nature of equilibrium (with or without cross-subsidization between contracts).

- Our starting point is **not** an *ex ante* institutional distinction between corporate forms (stock insurers and mutuals). We consider an insurance market where insurers (entrepreneurs) trade with risk adverse insurance seekers and possibly with risk neutral capitalists.

- **The nature of contracts, and thus the corporate form, are endogenous**. If an insurer only trades with insurance seekers by offering them participating contracts, we may call it a mutual. If an insurer offers non-participating contracts to insurance seekers and transfers profits to capitalists, it is a stock insurer.

- Main intuition of the paper : when second-best efficiency requires cross-subsidization between risk types, participating contracts act as an implicit threat againts deviant insurers who would like to attract low risks only.

- The paper predicts that individuals should be pooled in subgroups with cross-subsidization within the subgroups, and no cross-subsidization between subgroups (as in Spence, 1978). Participating contracts allow subgroups that include more than one type to be robust to competitive attacks.

- Prediction: the variance of the loss ratio between contracts should be larager for mutuals than for stock insurers.

Notations

$$Eu = (1-\pi)u(W_N - k + D) + \pi u(W_A + x + D)$$

where

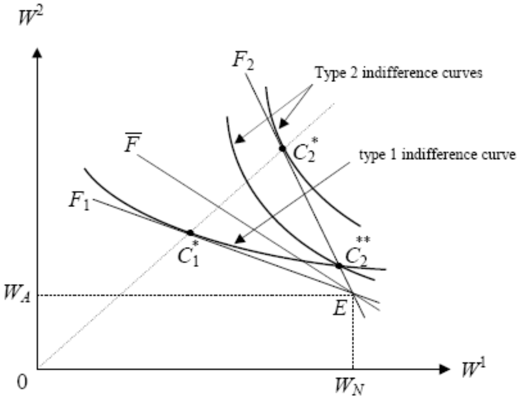
$$\begin{aligned}
u' &> 0, u'' < 0, \\
\pi &\in (0,1) = \text{ probability of an accident,} \\
W_N, W_A &= \text{ wealth without (with) accident,} \\
A &= W_N - W_A = \text{ loss in case of an accident,} \\
k &= \text{ insurance premium} \\
x &= \text{ net indemnity,} \\
D &= \text{ policy dividend.}
\end{aligned}$$

$D = 0$ for a non-participating policy (stock insurer); $D \neq 0$ depends on the insurer's profit for a participating policy (mutual). D is deterministic because of the law of large numbers.

Notations

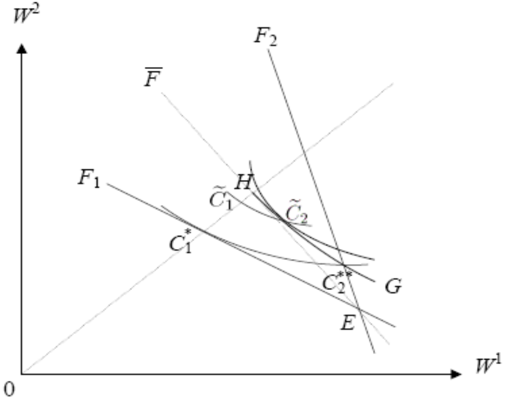
$$\begin{aligned}
W^1 &= W_N - k + D = \text{ final wealth if no accident,} \\
W^2 &= W_A + x + D = \text{ final wealth in case of an accident,} \\
\pi &\in \{\pi_1, \pi_2, ..., \pi_n\} \text{ with } 0 < \pi_n < \pi_{n-1} ... < \pi_1 < 1, \\
\lambda_i &= \text{ proportion of type } i \text{ individuals, with } \sum_{i=1}^{n} \lambda_i = 1.
\end{aligned}$$

The Rothschild-Stiglitz equilibrium



Nonexistence of equilibrium in the RS model

Equilibrium doesn't exist when $\lambda_1 \leq \lambda^{**}$



Extended RS-model

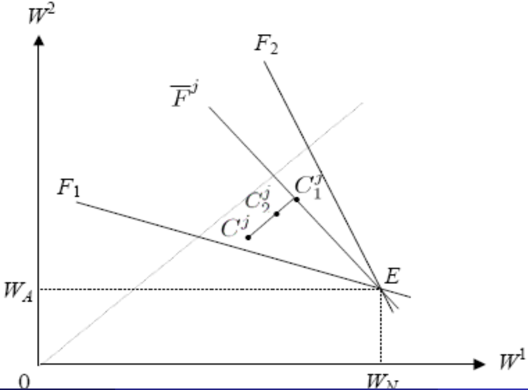
We characterize a subgame perfect equilibrium of a two stage game, with m insurers and a continuum of individuals with types $i = 1, ..., n$:

Stage 1: each insurer $j = 1, ..., m$ offers a menu of contracts,
Stage 2: individuals respond by choosing the contract they prefer among the offers of the insurers.

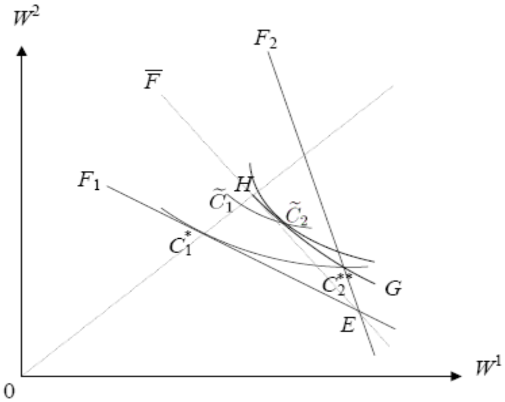
This is called the **extended RS-model**, because we allow insurers to offer either participating or non-participating contracts, while RS restrict attention to non-participating contracts. For a given individual, the attractiveness of a participating contract depends on the distribution of types of the other purchasers of the same contract, hence a participating contract may act as an implicit threat to deter competitive attacks.

Illustration in a simple case

Insurer j offers contract $C^j = (k^j, x^j)$ with policy dividend $D^j = \gamma^j P^j$, where $P^j =$ profit per policyholder and $\gamma^j \in [0,1]$. $E\overline{F}^j =$ average fair-odds line corresponding to the distribution of types among individuals who purchase C^j. C^j generates the lottery C_1^j if $\gamma^j = 1$ or C_2^j if $\gamma^j \in (0,1)$.



The two type case



Proposition 1

An equilibrium always exists in the extended RS model with $n = 2$; it is generically unique and it coincides with the MSW allocation. When $\lambda_1 \geq \lambda^{**}$, the separating contracts of the RS model C_1^*, C_2^{**} are offered at equilibrium without cross-subsidization and they may be participating or non-participating. When $\lambda_1 \leq \lambda^{**}$, the separating contracts $\widetilde{C}_1$, $\widetilde{C}_2$ are offered at equilibrium with cross-subsidization. Contract $\widetilde{C}_1$ which is chosen by type 1 individuals is participating, while $\widetilde{C}_2$ which is chosen by type 2 individuals may be participating or non-participating. The menu of contracts offered at an equilibrium with cross-subsidization maximizes the type 2 expected utility under the zero-profit constraint and incentive compatibility conditions.

The n type problem

Strategy of insurer j:
Menu of n contracts $= C^j = (C_1^j, C_2^j, ..., C_n^j, D^j(.))$ where $C_h^j = (k_h^j, x_h^j)$,
Policy dividend strategy $= D^j(.) = (D_1^j(.), ..., D_n^j(.))$, with
$D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j)$ for contract C_h^j

where

N_h^j = number of individuals who choose C_h^j,
P_h^j = profit per policyholder for C_h^j.

C^j is **fully participating** if

$$\sum_{h=1}^{n} N_h^j D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j) \equiv \sum_{h=1}^{n} N_h^j P_h^j$$

while $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j) \equiv 0$ for all h when C^j is **non-participating**

Candidate equilibrium allocation

A sequence of reservation expected utility levels $\overline{u}_i$ from Spence (1978): problem $\mathbb{P}_1$

$$\overline{u}_1 = \max(1-\pi_1)u(W^1) + \pi_1 u(W^2)$$
with respect to W^1, W^2, subject to
$$(1-\pi_1)W^1 + \pi_1(W^2 + A) = W_N$$

and for $2 \leq i \leq n$, problem $\mathbb{P}_i$

$$\overline{u}_i = \max(1-\pi_i)u(W_i^1) + \pi_i u(W_i^2)$$
with respect to $W_h^1, W_h^2, h = 1, ..., i$, subject to
$$(1-\pi_h)u(W_h^1) + \pi_h u(W_h^2) \geq \overline{u}_h \text{ for } h < i,$$
$$(1-\pi_h)u(W_h^1) + \pi_h u(W_h^2) \geq (1-\pi_h)u(W_{h+1}^1) + \pi_h u(W_{h+1}^2)$$
for $h < i$,
$$\sum_{h=1}^{i} [(1-\pi_h)W_h^1 + \pi_h(W_h^2 + A)] = W_N.$$

When $n = 2$, the optimal solution to $\mathbb{P}_2$ is the Miyazaki-Wilson equilibrium allocation.
$\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ = the optimal solution to $\mathbb{P}_n$.

Trade off: in $\mathbb{P}_n$, it is costly to increase the h - type expected utility EU_h above $\overline{u}_h$ for all $h < n$, but it relaxes the h - type incentive compatibility constraint.

When $n = 2$, the trade-off tips in favor of increasing EU_1 above $\overline{u}_1$ if $\lambda_1 < \lambda^{**}$. In that case there is cross-subsidization between types in $\mathbb{P}_2$.

More generally, at an optimal solution to $\mathbb{P}_n$, risk types are pooled in subgroups, with cross-subsidization within the subgroups and no cross-subsidization between subgroups.

Lemma 1

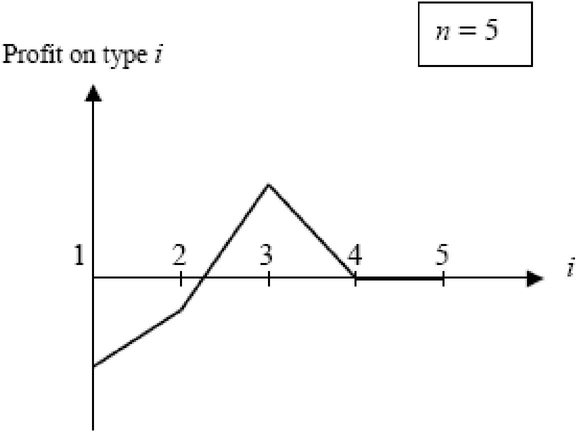
There exists $\ell_\theta \in \{0, ..., n\}$, $\theta = 0, ..., \overline{\theta} + 1$ with $\ell_0 = 0 < \ell_1 \leq \ell_2$ $... \leq \ell_{\overline{\theta}} < \ell_{\overline{\theta}+1} = n$ such that for all $\theta = 0, ..., \overline{\theta}$

$$\sum_{i=\ell_\theta+1}^{h} \lambda_i [W_N - (1-\pi_i)\widehat{W}_i^1 - \pi_i(\widehat{W}_i^2 + A)] \quad < \quad 0 \quad \text{for all } h = \ell_\theta + 1, ..., \ell_\theta$$

$$\sum_{i=\ell_\theta+1}^{\ell_{\theta+1}} \lambda_i [W_N - (1-\pi_i)\widehat{W}_i^1 - \pi_i(\widehat{W}_i^2 + A)] \quad = \quad 0.$$

Furthermore, we have

$$(1-\pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) \quad = \quad \overline{u}_i \text{ if } i \in \{\ell_1, \ell_2, ..., n\},$$
$$(1-\pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) \quad > \quad \overline{u}_i \quad \text{otherwise.}$$



$n = 5$
Profit on type i
1
2
3
4
5
i

Lemma 2

There does not exist any incentive compatible allocation $\{(W_h^1, W_h^2), h = 1, ..., n\}$ such that

$$(1 - \pi_{\ell_\theta})u(W_{\ell_\theta}^1) + \pi_{\ell_\theta}u(W_{\ell_\theta}^2) \geq \overline{u}_{\ell_\theta} \text{ for all } \theta = 1, ..., \overline{\theta} + 1$$

and

$$\sum_{h=1}^{n} \lambda_h [W_N - (1 - \pi_h)W_h^1 - \pi_h(W_h^2 + A)] > 0.$$

Proposition 2

$\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ is an equilibrium allocation sustained by a symmetric equilibrium of the market game where insurers offer participating contracts. Type i individuals choose $C_i^* = \widehat{C}_i \equiv (\widehat{k}_i, \widehat{x}_i)$ with $\widehat{k}_i = W_N - \widehat{W}_i^1, \widehat{x}_i = \widehat{W}_i^2 - W_A$ and $D^*(.)$ is such that

$$\sum_{i=1}^{n} D_i^*(N_1, P_1, ..., N_n, P_n) \equiv \sum_{i=1}^{n} N_i P_i$$

$$D_i^*(\frac{\lambda_1}{m}, \Pi(\widehat{C}_1), ..., \frac{\lambda_n}{m}, \Pi(\widehat{C}_n)) = 0 \text{ for all } i = 1, ..., n$$

$$D_{\ell_\theta}^*(N_1, P_1, ..., N_n, P_n) \equiv 0 \text{ for all } \theta = 1, ..., \overline{\theta} + 1.$$

Intuition of Proposition 2

Consider the allocation induced by $C^{j_0} \neq C^* = (C_1^*, ..., C_n^*)$ offered by a deviant insurer j_0. It corresponds to a compound lottery that mixes C^{j_0} and C^* but with the same profit as C^{j_0} alone, because insurers $j \neq j_0$ offer full participating contracts (they do not provide positive residual profits whatever the risk types of policyholders). Furthermore, all types ℓ_θ get at least the same expected utility as in the equilibrium allocation. Lemma 2 shows that this allocation cannot be profitable, hence deviant insurer j_0 does not make positive profit.

Deferred premium variations

- Reserves as a shock absorber : mutuals may shift the payment of underwriting profit to their members by transferring current profit to reserves and by later increasing or decresing premiums according to the level of accumulated surplus.
- Deferred premium variations are substitutes to policy dividends or supplementary calls.
- Paper also includes an extension to an overlapping generation setting, where individuals live for two periods.
- If transaction costs prevent individuals from changing their insurers between periods 1 and 2: no qualitative change in the results.
- Otherwise, more complex because moving to another insurer may signal risk type.

Concluding comments

- Initial motivation of the paper: an inquiry on the nonexistence of equilibrium in the RS model, starting with the observation that R&S restrict the set of insurance contracts to non-participating policies.
- Result is striking: Removing this restriction guarantees the existence of equilibrium. The equilibrium allocation coincides with the MSW allocation, but the underlying game form coincides with the RS model.
- Participating policies (or deferred premium variations in a dynamic setting) act as an implicit threat which prevents deviant insurers to attract low risk individuals only.

- A new explanation about why mutuals are so widespread in insurance markets and why they coexist with stock insurers, besides other explanations (reduction in agency costs, risk screening device or ability to cover undiversifiable risks).

- Our main conclusion is that mutuals are robust to competitive attacks in insurance markets with adverse selection, which may not be the case for stock insurance companies.