Kastatrophies for Rennes

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Catastrophes’ problems

- Very high possible losses
- With unknown very little likelihoods
- In an unkown maybe very remote future
- Whether we make precise what’s in red
- Or we take the problems more globally
- Whether we potter what we know how to do
- Or we develop adapted theories

Two basic reference theories

- Competitive markets, Finance:
- No reference to probas on events:
  risk measures from stat. on market prices
- Re-insurance: design of hedging contracts
- Individual decision theory
- EUT and extensions to ambiguity
- In Finance « real investments »
- EUT and extension to extreme events and remote future
- Direct applications of Insurance to non too catastrophic risks

First approach: competive markets and design of hedging financial instruments

Necessary cooperation between private and public sectors

- Public sector imposes regulations and supervises private initiatives. Experts decide on a catastrophe level
- Public sector manages prevention devices
- Insurance Cies measure damages and propose contracts (often subsidized)
- They are hedged by collective funds
- They get re-insured by private and collective organizations of hedging instruments (cat-bond markets)

Cat. bonds

- Because of regulation and tax concerns, a specific offshore structure called a "Special Purpose Vehicule" (SPV) proceeds to the bonds' emission.
- The SPV offers a reinsurance contract to the insurance companies seeking for one, at a cost.
- The total costs' amount is invested into Treasury Bills at a riskless rate, for a part,
- and into a short term securities' portfolio, for the remaining part.
- The riskless rate investments are meant to warrant the pledge on the investors,
- the short term high risky rate portfolio aims at hedging potential reinsured damage claims.

The market

- Seen from the insurer's point of view, the basis risk value is a fundamental argument in favour of bonds.
- Investors: claims contingent on catastrophes for diversifying their portfolios: zero, or close to zero, correlation

	Cat-bonds	**S&P 500**	**Treasury Bills**
Cat-bonds	-	-	-
S&P 5000	– 0.13	-	-
Treasury Bills	– 0.07	0.40	-

The Parametric Earthquake Catastrophe Bond (from Swiss-Re Media information)

- Negociations between Federal Mexico and local States and
- Insurance and re-insurance cies
- physical hazards are studied with precision so the financial side can design contracts and adapted premiums.
- public funds had to deal with insurance companies and be hedged by some financial instruments: notably specific cat-bonds.
- cat-bond on seism risk for Mexico: default has been related directly to the seism magnitude on the Richter scale, one trigger point magnitude 8.0.

Magnitude	7.0	7.5	8.0
% bond loss inner zone	40	100	-
% bond loss outer zone	20	60	100

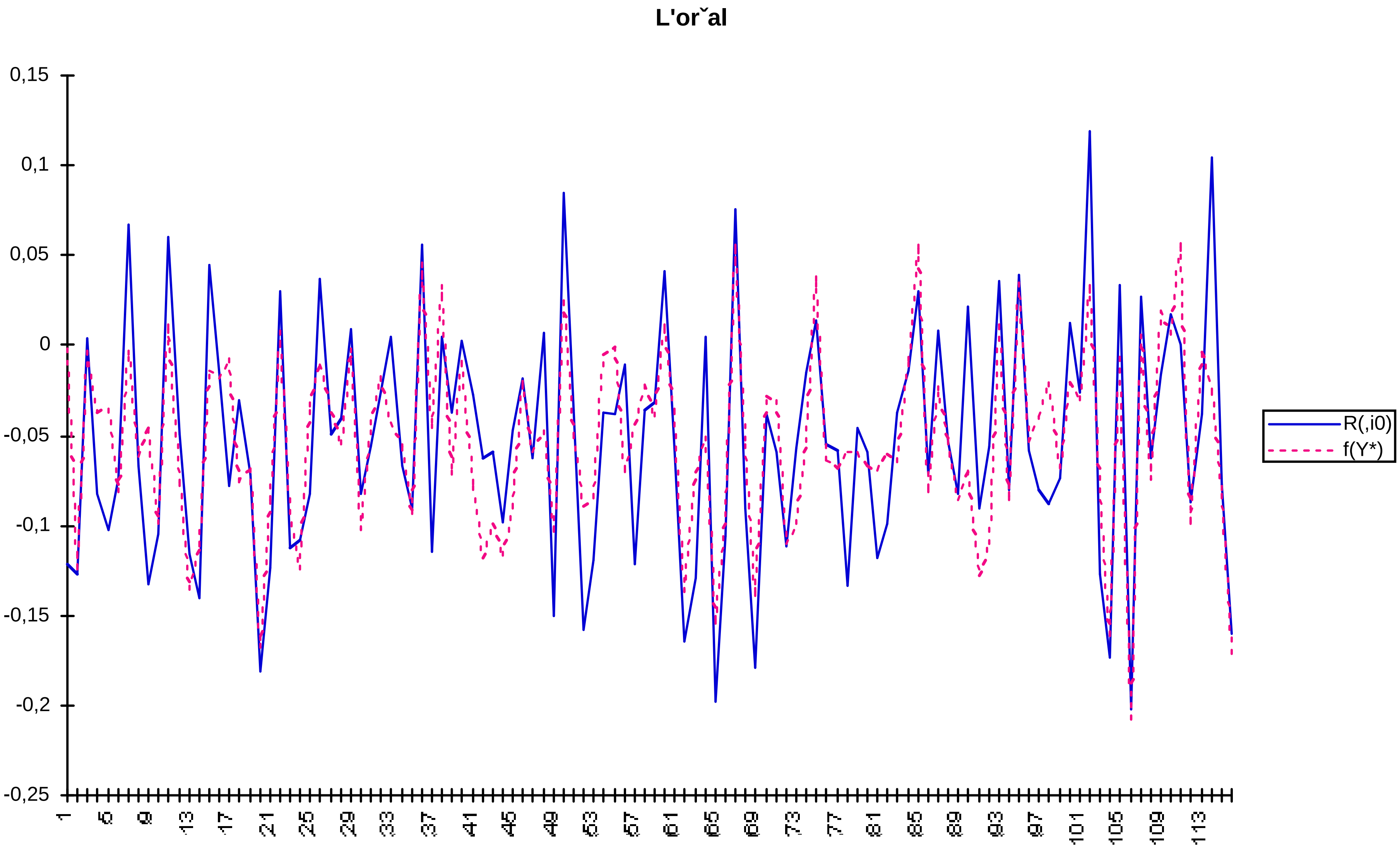
- the returns of the bonds, according to their ratings that go from BB to B- can be fixed so as to represent the price of risks.
- For instance if the issuer has a portfolio of riskless government bonds at 5% interest rate, plus a public fund risky bond,
- the total interest rate can be, say: 5+4 = 9% where 4% is the risk premium paid by the public fund.

Statistically replicating non tradable assets

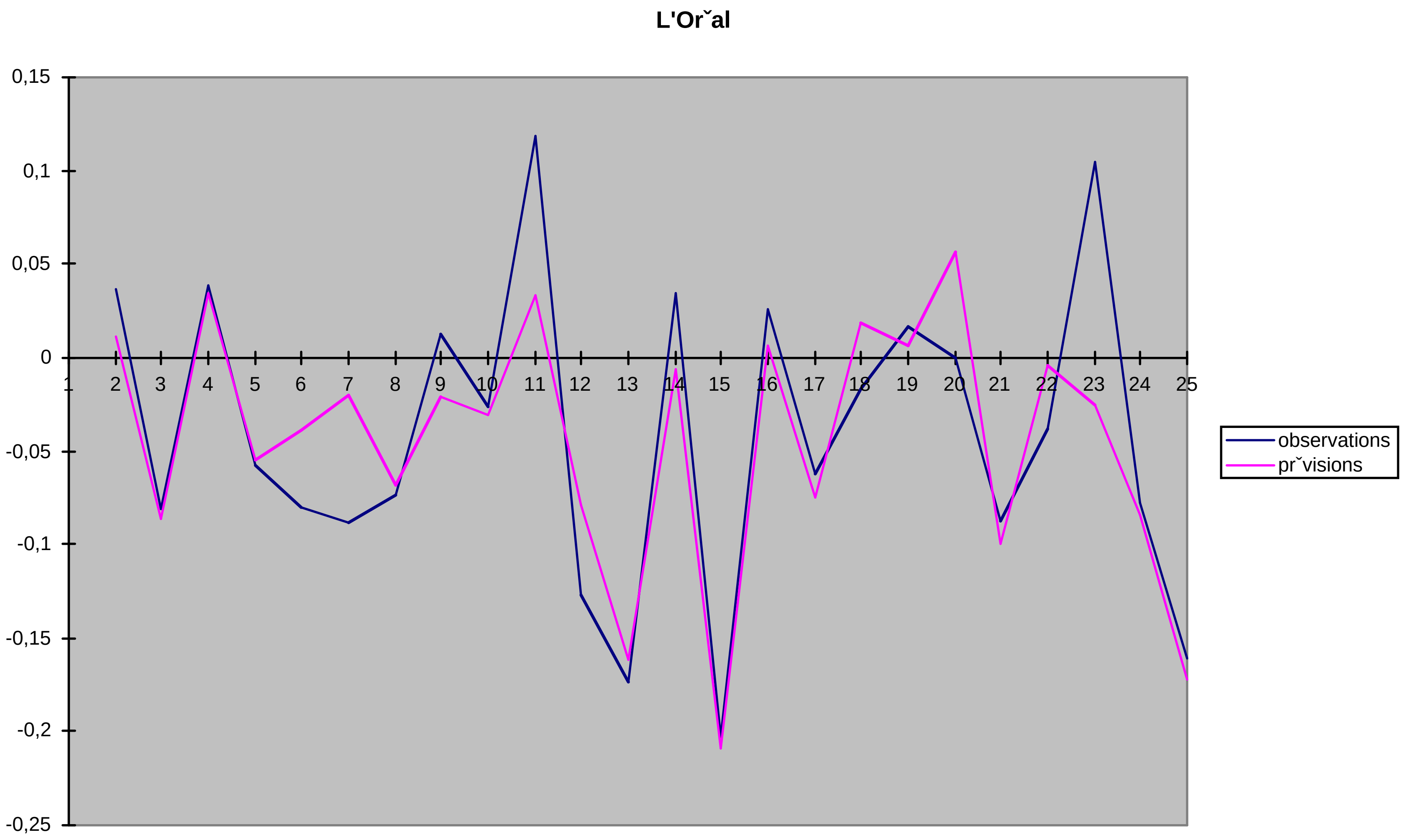
(from Kast, Lapied and Pardo)

- construct a portfolio of traded assets so as to maximise its functional correlation with the asset to be valued.
- The price process of the portfolio of traded assets yields the equivalent martingale measure from which any asset linearly dependent from the portfolio can be priced.
- Tested on traded assets for simplicity … but biased!

Replication of an asset (here a traded asset)



Confronting replication and forecasts



comonotonicity

- Two random variables X_1 and X_2 are comonotonic, if and only if there exists a third random variable, X_3 and two non decreasing functions g_1 and g_2 such that:
- $X_1 = g_1(X_3)$ and $X_2 = g_2(X_3)$
- $X_2 = g_2 o g_1^{-1}(X_1)$ and X_2 can be considered as a derivative asset of X_1.
- Binomial tree. No probability distribution.
- If Z is comonotonic with *X*, then the same binomial tree applies to *Z* and to *X* rates of returns.
- Cox, Ross, Rubinstein (1979) the binomial formula yields a risk adjusted probability and doesn't refer to a known probability distribution
- Chose a portfolio of marketed assets such that it minimises the difference between 1 and its Kendall coefficient with the risk
- The programming method makes a detour by an approximation methods such as genetic algorithms

Second approach: genreralized individual decision theory

Real assets valuation with ambiguity Kast and Lapied 2009 from Epstein and others 2002, 2006…

- Axiomatic approach in order to take ambiguity in a binomial tree:
- Up with capacity C<1/2 and Down with capacity C
- Yields a dynamically consistent Choquet random walk that converges toward a Brownian motion
- Results: deformations of both the mean and the variance in our model of Choquet expectations.

Theoretical 1: a global vision of the future (Kast and Lapied 2008)

- The future: payoffs processes contingent on SxT (finite)
- Axiomatic valuation consistent with information arrivals at times t in T.
- Ghirardato-Fubini (1998) theorem limitatations:
- Whether RV are comonotonic (EU) or
- trajectories are time sepable (additive discount factors)
- Results: updating capacities rules (≠ from Bayes)
- « uspstating » discount factors rule (hard to interpret!)

Theroretical 2: extreme events and time horizon (Chichilnisky 1998, 2009)

- Advantage: keeps linear valuation and easy optimal techniques
- However doesn’t satisfy time consistency
- **Chichilnisky long term criterion:**

A compound of EU on a finitely countable additive proba

And a limit utility for limit to infinity dates

- **Chichilnisky topology of fear (towards extreme events)**
- A compound of EU on a countable additive probability (usual events)
- And a expected utility on a discrete bounded measure for extreme (feared) events.
- Problem: how to determine the compound factors, i.e. what is extreme?

Some econometrics?
Classical insurance?

Back on earth: non too catatrophic floods and other climate risks for agriculture (Kast Enjolras and Sentis 2009)

- The development of crop insurance policies:
  - From public coverage to private coverage.
  - From single-crop policies to multi-crop policies.

- A market with regular growth but many limitations:
  - High degree of subsidization.
  - Incomplete markets → Inefficient coverage.
- Risk covered: Individual, Systematic or Both?
- Form: Insurance policies or Financial contracts?
- Level of reference: Individual, Regional or National?
- Strategy: n*1 crops or 1 portfolio of n crops?

The econometric model

- The Linear Additive Model (LAM) describes a relationship between individual yield and the mean yields in a given area for a given crop.

$$\tilde{y}_i = E(\tilde{y}_i) + \beta_i(\tilde{y} - E(\tilde{y})) + \tilde{\varepsilon}_i$$

- *y_i is the individual yield, y is the area yield, β_i is the sensitivity of the individual crop yield to the movements of the crop area-yield and ε_i is the residual of the model.*
- **Multi-LAM:** ***j crops, j estimates***

$$\tilde{y}_i = \sum_j x_{ij}\left[E(\tilde{y}_{ij}) + \beta_{ij}(\tilde{y}_j - E(\tilde{y}_j))\right] + \tilde{\varepsilon}_i$$
$$\text{with } \sum_j x_{ij} = 1, \forall i$$

Application to agricultural data base

- Aim: Test the validity of the Multi-LAM compared to the other approaches (Additive LAM and Farm LAM).
- Survey of French farmers (FADN): accounting data from 1990 to 2006, more than 2000 farms.
- Aggregated data (Agreste, Météo France).

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Characterization of insurance possibilities

- Use of the estimated β coefficients:
  - β coefficients estimate the sensibility of an individual yield to an area yield.
  - Analogy with financial theory (APT).
  - β = hedge ratios → Direct use for financial contracts.
- Cover full risk through the identification procured by the Multi-LAM:
  - Systematic risk optimized → financial contracts.
  - Individual risk reduced → (participating) insurance policies.

Morality

Catastrophes and other problems related to the precautionnary principle bring us back to face Uncertainty about the future.

- Most of our models in economics are based on scientific certainty (i.e. a probability measure around a mean considered as reliably certain).
- We need theories to understand what is Uncertainty and what is Time in order to tackle the problems we face. My understanding is that both can’t be separated in human sciences the way it is in other sciences
- We need practice also that can be developed in advance of theories

(the notion of risks and finance were developed before the word uncertainty was invented)

- Some of our available models can be extended and implemented to solve imperfectly well circonscripted problems
- But let’s not pretend they justify policies and practices, specially when their assumptions are not satisfied