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## « The Effect of Ambiguity Aversion on Self-insurance and Self-protection »

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# Motivation

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- Some risks are ambiguous (i.e., have « unknown » probability distributions)
- Does ambiguity justify more preventive efforts?
- Does ambiguity aversion reinforce risk aversion?

# Some results

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We distinguish self-insurance (reduction in the size of the loss) from self-protection (reduction in the probability of the loss)

- Ambiguity aversion always raises the **marginal cost of preventive efforts** (self-insurance + self-protection) under risk aversion
- Ambiguity aversion always raises the **marginal benefit of self-insurance** under risk aversion;

Moreover, this effect on marginal benefit dominates the previous effect on marginal cost, so that ambiguity aversion always raises self-insurance

- The effect of ambiguity aversion on the **marginal benefit of self-protection** is unclear;

But no effect in some regular cases, so that ambiguity aversion reduces self-protection due to the effect on marginal cost

# Ongoing research (no paper yet!)

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- Based on :

Alary, D., Gollier, C. and N. Treich, « The effect of ambiguity aversion on self-insurance and self-protection », work in progress.

See also Treich N., 2009, « The value of a statistical life under ambiguity aversion », *Journal of Environmental Economics and Management*, forthcoming.

# Outline

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1. Classical expected utility approach: The effect of risk aversion on self-insurance (SI) and on self-protection (SP)
2. Modeling ambiguity and ambiguity aversion
3. Willingness to pay for SI and SP under ambiguity aversion
4. More general results on the effect of ambiguity aversion

# Self-insurance under expected utility

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- Reduction of the size loss if an accident (e.g., earthquake, flood, fire) occurs – example sprinkler systems
- A simple self-insurance decision model under expected utility theory:

$$\text{Max}_e (1-p)u(w-e) + pu(w-e-L(e))$$

$p$ : probability of loss,  $e$ : self-insurance effort

$L(e)$ : loss, with  $L(.) > 0$ ,  $L'(e) < -1$ ,  $L''(.) > 0$

$w$ : initial wealth,  $u(.)$ : vNM utility function, with  $u'(.) > 0$  and  $u''(.) \leq 0$

# Optimal self-insurance effort

- First order condition

$$g(e) = -(1-p)u'(w-e) + p(-1-L'(e))u'(w-e-L(e)) = 0$$

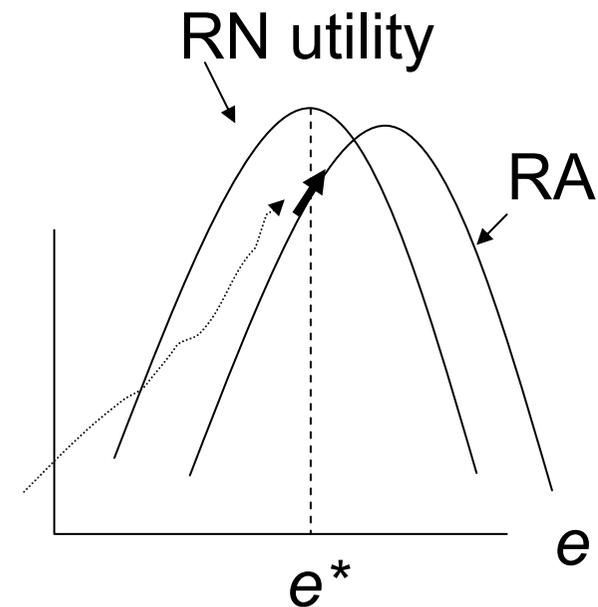
- Under risk neutrality

$$-1-pL'(e^*) = 0$$

- Risk aversion increases self-insurance iff  $g(e^*) \geq 0$

$$g(e^*) = (1-p)(u'(w-L(e^*)-e^*) - u'(w-e^*)) \geq 0$$

- Always true. Risk averse agents thus invest more in self-insurance than risk neutral agents



# Self-protection under expected utility

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- Reduction of the probability of the loss
- A simple self-protection decision model under expected utility theory:

$$\text{Max}_e (1-p(e))u(w-e) + p(e)u(w-L-e)$$

with  $p(.) \in [0, 1]$ ,  $p'(.)<0$  and  $p''(.)>0$

- First order condition

$$g(e) = -p'(e)(u(w-e)-u(w-L-e)) - (1-p(e))u'(w-e) - p(e)u'(w-L-e) = 0$$

# Risk aversion and self-protection

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- Risk aversion may decrease, and not increase, self-protection (Dionne and Eeckhoudt, 1985)
- Depends on whether  $p$  is lower or larger than  $\frac{1}{2}$ , and on conditions on  $u(\cdot)$  (Ehrlich and Becker, 1972; Darchaoui et al., 2004; Eeckhoudt and Gollier, 2005)
- Intuition:  $\frac{1}{2}$  is the critical probability threshold below which a reduction in probability reduces the variance of the risk

# Ambiguity

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- Probability of loss is « unknown »
- Compare for instance two situations:
  - i) the probability of loss is known  $p=1/3$ , or
  - ii) the probability of loss  $P$  is unknown, but with « expectation »  $EP=1/3$  (to keep the risk of same magnitude)
- Under ambiguity neutrality (i.e., expected utility), the objective is linear in probability, and the two situations are equivalent
- Under ambiguity theories, the two situations are not equivalent

# Ellsberg's suggested experiment

Act	30 red balls	black balls	yellow balls
X	\$W	0	0
Y	0	\$W	0
X'	\$W	0	\$W
Y'	0	\$W	\$W

Many people would choose  $X > Y$  and  $Y' > X'$   
Inconsistent with Savage's theory of a unique subjective probability

# Ambiguity aversion

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- Experimental evidence (e.g., Sarin and Weber, 1993, Halevy, 2007) and survey data (e.g., Viscusi, 1998, Chesson and Viscusi, 2003)
- Some market evidence (e.g., Camerer and Weber, 1992)
- Explain some empirical puzzle (e.g., in finance, Mukerji and Tallon, 2001, Chen and Epstein, 2002)
- But difficult to estimate ambiguity and ambiguity aversion, and introduce new anomalies, especially for intertemporal decision-making (e.g., Al-Najjar and Weinstein, 2009)

# Why would individuals be ambiguity averse?

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- Open question
- Using brain imaging, Camerer et al. (2007) find evidence that the amygdala is more active under ambiguity conditions, and notice that the « *amygdala has been specifically implicated in processing information related to fear* » (Camerer et al., 2007, p. 131)
- Emotional fearful situations, like life-threatening situations, lead individuals to react to probabilities and outcomes in a manner that is very different from that postulated by expected utility (Loewenstein, 2007)

# Some models with « ambiguity aversion »

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- Let  $V(p) = (1-p)u(w) + pu(w-L)$  the EU conditional on  $P=p$
- Gadjos-Takashi-Tallon-Vergnaud (2008) (see also, Maccheroni-Marinacci-Rustichini, 2006):

$$(1-\mu) EV(P) + \mu V(p_{high})$$

in which  $\mu$  is the uncertainty aversion parameter

- Klibanoff-Marinacci-Mukerji (hereafter KMM) (2005)

$$\emptyset^{-1}[E\emptyset[V(P)]]$$

in which  $\emptyset$  concave is equivalent to ambiguity aversion

# Polar case: Risk elimination

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- From now, I will assume KMM preferences
- Risk elimination is an ultimate form of self-insurance (where the loss equals zero) or self-protection (where the probability of loss equals zero)
- Let  $C$  the WTP for risk elimination under ambiguity aversion

$$U(w-C) = \emptyset^{-1}[E\emptyset[V(P)]] \leq \emptyset^{-1}[\emptyset[EV(P)]] = V(EP) = u(w-C_0)$$

with  $V(p) = (1-p)u(w) + pu(w-L)$ , and where  $C_0$  is **WTP** under EU

- Therefore ambiguity aversion raises WTP for risk elimination
- Intuition: eliminating the risk also eliminates the ambiguity

# WTP for self-insurance under KMM

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- WTP  $C(e)$  for loss reduction  $e$

$$E\emptyset[(1-P)u(w-C(e))+Pu(w-C(e)-L+e)] = E\emptyset[V(P)]$$

- WTP for infinitesimal self-insurance

$$C'(0) = u'(w-L) \hat{E}P / ((1-\hat{E}P)u'(w) + \hat{E}P u'(w-L))$$

where  $\hat{E}P = E(P\emptyset'[P])/E\emptyset'[P] > EP$  under  $\emptyset$  concave (Monotone likelihood ratio)

- Two effects: ambiguity increases expected marginal cost (denominator) but increases expected marginal benefit (numerator) of self-insurance
- The second effect dominates: ambiguity aversion increases the WTP for self-insurance

# WTP for self-protection under KMM

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- WTP  $C(e)$  for a reduction  $e$  in the probability of loss

$$E\emptyset[(1-P+e)u(w-C(e))+(P-e)u(w-C(e)-L)] = E\emptyset[V(P)]$$

- WTP for infinitesimal self-protection

$$C'(0) = (u(w) - u(w-L)) / ((1-\hat{E}P)u'(w) + \hat{E}P u'(w-L))$$

where  $\hat{E}P = E(P\emptyset'[P]) / E\emptyset'[P] > EP$  under  $\emptyset$  concave (Monotone likelihood ratio)

- One effect only: increase in the marginal cost (denominator)
- Ambiguity aversion therefore decreases WTP for self-protection
- Opposite effect of ambiguity aversion on self-insurance vs. self-protection

# A more general model

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$$\text{Max}_e \mathbb{E}^{-1}[E\mathbb{E}[(1-P(e))u(w-e) + P(e)u(w-e-L(e))]]$$

- Model combining both self-insurance and self-protection
- Model studying optimal preventive effort (not only polar cases)
- What's the effect of ambiguity aversion on optimal effort  $e$ ?
- Remark:  $P(e)$  is a random variable

# Results – Three distinct effects

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1. Ambiguity aversion always increases **expected marginal cost** of preventive measures ( $\downarrow e$ )
2. Ambiguity aversion always increases **expected marginal benefit of self-insurance** ( $\uparrow e$ ); moreover this effect dominates the first negative effect of marginal cost
3. Ambiguity aversion has an ambiguous effect on the **expected marginal benefit of self-protection**, depends on how  $P'(e)$  covaries with  $P(e)$

Intuition: ambiguity aversion leads to « focus » more on high probabilities of loss; thus the effect depends on whether self-protection efforts reduce high probabilities of loss

# Extensions?

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- Other ambiguity aversion models (seems to lead to similar results)
- More than two states (difficult: not surprising since limited results even under expected utility – see, e.g., Jullien, Salanié, Salanié, 1999)
- Comparative statics in KMM model: wealth, risk aversion, baseline probability of loss...
- Ambiguous loss  $L$
- Add insurance demand (but raises issues concerning the behavior of insurers)
- State-dependent preferences (Treich, 2009)

# Conclusions

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- Ambiguity aversion always increases self-insurance (and therefore reinforces the effect of risk aversion)
- Ambiguity aversion may well decrease self-protection; depends on the relative efficiency of the risk-reduction technology in the « bad states »
- Are there real life examples of these effects? Are ambiguity and ambiguity aversion important to understand individual preventive efforts?
- Policy implications? (tricky because ambiguity theories raise normative issues)